Abstractions from Proofs

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Software Model Checking Seminar SS05

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Outline

1. Introduction
   - Motivation
   - Craig Interpolation

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   - Completeness

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3. Language and Abstraction
   - Program representation
   - Strongest Postcondition and Predicate Abstraction

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3. Language and Abstraction
   - Program representation
   - Strongest Postcondition and Predicate Abstraction
4. Putting Things Together
   - Basic Definitions
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- However, most predicates are only locally useful.
- We need a way to
  - learn a minimal set of predicates from a spurious counterexample
  - use these predicates only where they are useful

Motivation

Example

```
      while(*) {
1:      if(p1) lock();
        if(p1) unlock();
        ...
2:      if(p2) lock();
        if(p2) unlock();
        ...
n:      if(pn) lock();
        if(pn) unlock();
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assume $p1$;
lock();
assume $\neg p1$;
assume $p2$;
lock();

Each pi is useful only between i and $i + 1$.

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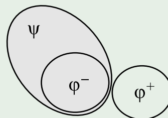

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ψ always exists if $\varphi^- \wedge \varphi^+$ unsatisfiable.

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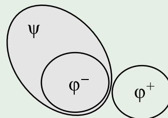

FOL

We consider formulas in quantifier-free first-order logic of linear equality:

- a **term** is of the form $c_0 + c_1 x_1 + ... + c_n x_n$ with variables $x_1, ..., x_n$ and integer constants $c_1, ..., c_n$

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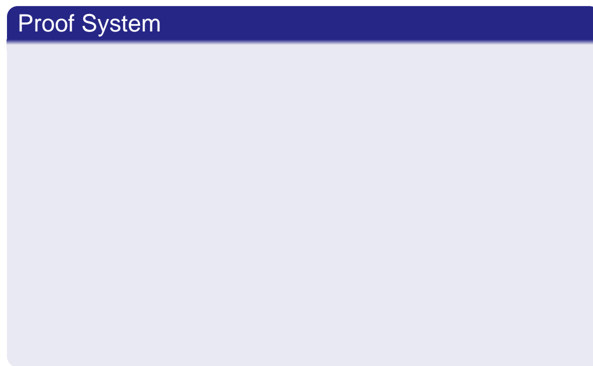
Interpretation of $\Gamma \vdash \Delta$: $\bigwedge_{\phi \in \Gamma} \phi \Rightarrow \bigvee_{\phi \in \Delta} \phi$

Refutation Proof System

Assume we have a theorem prover that generates refutations for sets of clauses using the following



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$$\textsc{Res}\ \frac{\Gamma \vdash \{\phi\} \cup \Theta \qquad \Gamma \vdash \{\neg\phi\} \cup \Theta'}{\Gamma \vdash \Theta \cup \Theta'}$$

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If $(\varphi^-, \varphi^+) \vdash \bot\ [\psi]$ then ψ is an interpolant for (φ^-, φ^+).

Derivation rules

Proof rule

$$\textsc{Hyp}\ \frac{}{\Gamma \vdash \phi}\ \phi \in \Gamma$$

Derivation rules

$$\textsc{Hyp-A}\ \frac{}{(\varphi^-, \varphi^+) \vdash \phi\ [\phi]}\ \phi \in \varphi^-$$

$$\textsc{Hyp-B}\ \frac{}{(\varphi^-, \varphi^+) \vdash \phi\ [\top]}\ \phi \in \varphi^+$$

$\top$ stands for $0 \leq 0$

Derivation rules

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$$\textsc{Comb}\ \frac{\Gamma \vdash 0 \le x \quad \Gamma \vdash 0 \le y}{\Gamma \vdash 0 \le c_1 x + c_2 y}\ c_{1,2} > 0$$

Derivation rule

$$\textsc{Comb}\ \frac{\begin{array}{c}(\varphi^-, \varphi^+) \vdash 0 \le x \ \ [0 \le x'] \\ (\varphi^-, \varphi^+) \vdash 0 \le y \ \ [0 \le y']\end{array}}{(\varphi^-, \varphi^+) \vdash 0 \le c_1 x + c_2 y \ \ [0 \le c_1 x' + c_2 y']}\ c_{1,2} > 0$$

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Derivation rule

$$\textsc{Contra}\ \frac{(\{a_1, \ldots, a_k\}, \{b_1, \ldots, b_m\}) \vdash \bot \ \ [\psi]}{(\varphi^-, \varphi^+) \vdash \neg a_1, \ldots, \neg a_k, \neg b_1, \ldots, \neg b_m \ \ [(\neg a_1 \vee \cdots \vee \neg a_k) \vee \psi]}$$

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$$\textsc{Res-A}\ \frac{\begin{array}{c}(\varphi^-, \varphi^+) \vdash \phi, \Theta \ \ [(\phi \vee \rho) \vee \psi] \\ (\varphi^-, \varphi^+) \vdash \neg\phi, \Theta' \ \ [(\neg\phi \vee \rho') \vee \psi']\end{array}}{(\varphi^-, \varphi^+) \vdash \Theta, \Theta' \ \ [(\rho \vee \rho') \vee (\psi \vee \psi')]}$$

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- We refer to ψ as $\text{ITP}.(\varphi^-, \varphi^+).\mathcal{P}$

Derivation rules

Example

Let $\varphi := \varphi^- \cup \varphi^+$ with

- $\varphi^- := (0 \leq y - x)(0 \leq z - y)$
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The Interpolant is generated by a linear scan of the proof.

Program representation

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  - $f(x_1, \ldots, x_n)$, function call with actual parameters $x_1, \ldots, x_n$
  - and `return` to the caller

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A **CFA** is defined by $C_f = (L_f, E_f, l_f^0, Op_f, V_f)$ with

- $L_f \subseteq PC, \quad E_f \subseteq L_f \times L_f, \quad l_f^0 \in L_f$
- $Op_f : E_f \rightarrow Ops$
- local variables $V_f \subseteq Lvals$ with
  - $X_f \subseteq V_f$ set of formal parameters
  - $r_f \in V_f$ variable storing return value

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- a **trace** of $\mathbb{P}$ is a sequence $(op_1 : pc_1); \ldots ; (op_n : pc_n)$ such that
  - $Op(l^0_{main}, pc_1) = op_1$
  - function calls and returns are properly matched
  - operations remaining within a function body respect paths in the corresponding CFA

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- The **predicate abstraction** of φ w.r.t. P is the strongest formula ψ such that
  - ψ is a boolean combination of predicates in P and
  - φ implies ψ

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- We will consider classes of programs of increasing complexity:
  - **PI**: flat and pointer-free
  - **PII**: pointer-free programs
  - **PIII**: flat programs with pointers
  - **PIV**: general programs (pointers and function calls)

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- a **constraint map** is a function $\Gamma : \mathbb{N} \to FOL$ that maps each point i of trace t to a constraint $\Gamma.i$

Basic Definitions

We need the following notions:

- an **lvalue map** is a function $\theta \in Lvm = Lvals \rightarrow \mathbb{N}$
  - the operator **Upd** : $Lvm \rightarrow 2^{Lvals} \rightarrow Lvm$ is defined as follows:

    $$(Upd.\theta.L).l = \begin{cases} \theta.l & \text{if } l \notin L, \\ i_l & \text{otherwise.} \end{cases}$$

    with fresh integer i_l
- **Sub**$.\theta.l = \langle l, \theta.l \rangle$
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- a **constraint map** is a function $\Gamma : \mathbb{N} \rightarrow FOL$ that maps each point i of trace t to a constraint $\Gamma.i$
- $\bigwedge_{1 \leq i \leq n} \Gamma.i$ is the **trace formula** (TF) of trace $t = (op_1 : pc_1); \ldots; (op_n : pc_n)$
  The trace is feasible iff the TF is satisfiable.

Basic Definitions

For each class of programs, we will define

- an operator **Con**, with $\mathit{Con}.(\theta,\Gamma).t = (\theta',\Gamma')$ that updates the lvalue map θ and computes a new constraint map Γ' given a trace t

Algorithm *Extract*

Idea: t is *SP*-infeasible iff t is SP_{Π}-infeasible for $\Pi = \mathit{Extract}.t$

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Algorithm *Extract*

Example

trace t	$Con.(\theta_0, \Gamma_0).t$	
$(\text{assume}(b > 0) : pc_1);$	$\langle b, 0\rangle > 0$	φ^-
$(c := 2 * b : pc_2);$	$\langle c, 1\rangle = 2 * \langle b, 0\rangle$	
$(a := b : pc_3);$	$\langle a, 2\rangle = \langle b, 0\rangle$	
$(a := a - 1 : pc_4);$	$\langle a, 3\rangle = \langle a, 2\rangle - 1$	
$(\text{assume}(a < b) : pc_5);$	$\langle a, 3\rangle < \langle b, 0\rangle$	φ^+
$(\text{assume}(a = c) : pc_6)$	$\langle a, 3\rangle = \langle c, 1\rangle$	

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PI programs

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$(x := e : pc_i)$

$SP.\varphi.t$:

$SP_\Pi.\varphi.t$:

$Con.(\theta, \Gamma).t$:

PI programs

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$SP.\varphi.t$: $\exists x'.(\varphi[x'/x] \wedge x = e[x'/x])$
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PI programs

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This allows us to express the effect of f in terms of its arguments, at the call site.

PII programs

Trace

$$(y := f(\vec{e}) : pc_i);$$
$$t_1;$$
$$(\text{return} : pc_j)$$

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PII programs

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$$\begin{aligned} SP.\varphi.t: \quad & \exists y', \vec{\phi}.\varphi[y'/y] \wedge \\ & \exists r.\vec{\phi} = \vec{e}[y'/y] \wedge \\ & \exists V_f^-.SP.true.t_1 \wedge \\ & y = r \end{aligned}$$

with fresh y', formals $\vec{x}$, return var r, $V_f^- = V_f \backslash (sym_f \cup \{r\})$

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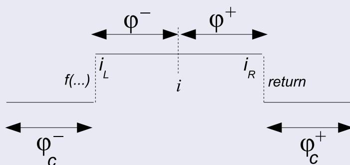
$Con.(\theta, \Gamma).t$: (θ', Γ')
where

$$\theta' = Upd.\theta.\{y\}$$
$$\theta_I = Upd.\theta'.V_f$$
$$\Gamma_I = \Gamma[i \mapsto Sub.\theta_I.\vec{\phi} = Sub.\theta_I.\vec{e}]$$
$$(\theta_O, \Gamma_O) = Con.(\theta_I, \Gamma_I).t_1$$
$$\Gamma' = \Gamma_O[j \mapsto Sub.\theta'.y = Sub.\theta_O.r]$$

Getting Well-Scoped Predicates

Scope of symbols



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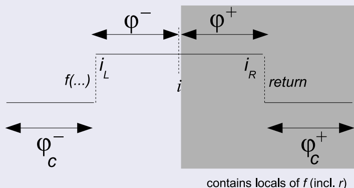

To get predicates that contain only symbols that are in scope at location i, we construct the interpolant of $(\varphi^-, \varphi^+ \wedge \varphi_C^- \wedge \varphi_C^+)$

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Handling Pointers

- Theory of arrays defines functions **sel**, **upd** and the axiom

$$sel(upd(M, a, v), b) = \begin{cases} v & \text{if } a = b, \\ sel(M, b) & \text{otherwise.} \end{cases}$$

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- We use M as a map from lvalues to memory expressions over M.

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  - **Reach.X** $= \{*^k x \mid x \in X \textit{ and } k \geq 0\}$

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  - `assume`$(\bigwedge_{l \in Reach.X_f} l = \phi_l)$

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  - the callee starts with a copy of the caller's store
  - upon return, the caller refreshes his store using the callee's store
- No explicit return variable, instead update some cell passed as argument

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- We require that every cell of the caller which is unreachable from the parameters remains unchanged after a call.

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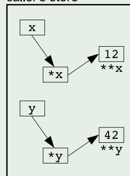
- Upon return, we copy into the caller's store the contents of the callee's store that are reachable from the passed parameters.

PIII and PIV programs

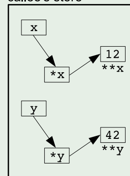
Example

Function call:

caller's store



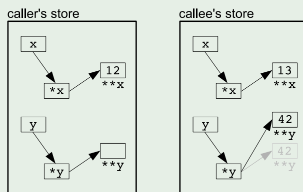
callee's store



PIII and PIV programs

Example

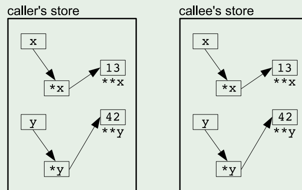
Upon return:



PIII and PIV programs

Example

Refresh caller’s store:



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Example

$(*y := 0 : pc_1);$
$(\text{inc}(y) : pc_2);$

$\langle *\langle y, 0\rangle, 1\rangle = 0$
$\langle \phi_x, 1\rangle = \langle y, 0\rangle \wedge \langle \phi_{*x}, 1\rangle = \langle *\langle y, 0\rangle, 1\rangle$

PIII and PIV programs

Example

$(*y := 0 : pc_1);$
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$\langle *\langle y, 0\rangle, 1\rangle = 0$
$\langle \phi_x, 1\rangle = \langle y, 0\rangle \wedge \langle \phi_{*x}, 1\rangle = \langle *\langle y, 0\rangle, 1\rangle$
$\quad\langle x, 4\rangle = \langle \phi_x, 1\rangle$
$\quad\wedge\langle *\langle x, 4\rangle, 5\rangle = \langle *\langle \phi_x, 1\rangle, 7\rangle$
$\quad\wedge\langle *\langle x, 4\rangle, 5\rangle = \langle \phi_{*x}, 2\rangle$

PIII and PIV programs

Example

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$(\text{inc}(y) : pc_2);$
$\quad(\mathit{assume}(x = \phi_x$
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$\quad(*x := *x + 1 : pc_4);$

$\langle *\langle y, 0\rangle, 1\rangle = 0$
$\langle \phi_x, 1\rangle = \langle y, 0\rangle \wedge \langle \phi_{*x}, 1\rangle = \langle *\langle y, 0\rangle, 1\rangle$
$\quad\langle x, 4\rangle = \langle \phi_x, 1\rangle$
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$\quad\wedge\langle *\langle x, 4\rangle, 5\rangle = \langle \phi_{*x}, 2\rangle$
$\langle *\langle x, 4\rangle, 6\rangle = \langle *\langle x, 4\rangle, 5\rangle + 1$
$\quad \mathit{ite}.(\langle x, 4\rangle = \langle \phi_x, 1\rangle)$
$\qquad .(\langle *\langle \phi_x, 1\rangle, 8\rangle = \langle *\langle x, 4\rangle, 5\rangle + 1)$
$\qquad .(\langle *\langle \phi_x, 1\rangle, 8\rangle = \langle *\langle \phi_x, 1\rangle, 7\rangle)$

PIII and PIV programs

Example

Program	Constraints
$(*y := 0 : pc_1)$;	$\langle *\langle y, 0\rangle, 1\rangle = 0$
$(\mathrm{inc}(y) : pc_2)$;	$\langle \phi_x, 1\rangle = \langle y, 0\rangle \wedge \langle \phi_{*x}, 1\rangle = \langle *\langle y, 0\rangle, 1\rangle$
$(assume(x = \phi_x$	$\langle x, 4\rangle = \langle \phi_x, 1\rangle$
$\wedge * x = \phi_{*x}) : pc_3)$	$\wedge\langle *\langle x, 4\rangle, 5\rangle = \langle *\langle \phi_x, 1\rangle, 7\rangle$
	$\wedge\langle *\langle x, 4\rangle, 5\rangle = \langle \phi_{*x}, 2\rangle$
$(*x := *x + 1 : pc_4)$;	$\langle *\langle x, 4\rangle, 6\rangle = \langle *\langle x, 4\rangle, 5\rangle + 1$
	$ite.(\langle x, 4\rangle = \langle \phi_x, 1\rangle)$
	$.(\langle *\langle \phi_x, 1\rangle, 8\rangle = \langle *\langle x, 4\rangle, 5\rangle + 1)$
	$.(\langle *\langle \phi_x, 1\rangle, 8\rangle = \langle *\langle \phi_x, 1\rangle, 7\rangle)$
$(return : pc_5)$;	$ite.(\langle y, 0\rangle = \langle \phi_x, 1\rangle)$
	$.(\langle *\langle y, 0\rangle, 9\rangle = \langle *\langle \phi_x, 1\rangle, 8\rangle)$
	$.(\langle *\langle y, 0\rangle, 9\rangle = \langle *\langle y, 0\rangle, 1\rangle)$

PIII and PIV programs

Example

$(*y := 0 : pc_1);$	$\langle *\langle y,0\rangle,1\rangle = 0$
$(\text{inc}(y) : pc_2);$	$\langle \phi_x,1\rangle = \langle y,0\rangle \wedge \langle \phi_{*x},1\rangle = \langle *\langle y,0\rangle,1\rangle$
$(assume(x = \phi_x$	$\langle x,4\rangle = \langle \phi_x,1\rangle$
$\wedge *x = \phi_{*x}) : pc_3)$	$\wedge\langle *\langle x,4\rangle,5\rangle = \langle *\langle \phi_x,1\rangle,7\rangle$
	$\wedge\langle *\langle x,4\rangle,5\rangle = \langle \phi_{*x},2\rangle$
$(*x := *x + 1 : pc_4);$	$\langle *\langle x,4\rangle,6\rangle = \langle *\langle x,4\rangle,5\rangle + 1$
	$ite.(\langle x,4\rangle = \langle \phi_x,1\rangle)$
	$.(\langle *\langle \phi_x,1\rangle,8\rangle = \langle *\langle x,4\rangle,5\rangle + 1)$
	$.(\langle *\langle \phi_x,1\rangle,8\rangle = \langle *\langle \phi_x,1\rangle,7\rangle)$
$(return : pc_5);$	$ite.(\langle y,0\rangle = \langle \phi_x,1\rangle)$
	$.(\langle *\langle y,0\rangle,9\rangle = \langle *\langle \phi_x,1\rangle,8\rangle)$
	$.(\langle *\langle y,0\rangle,9\rangle = \langle *\langle y,0\rangle,1\rangle)$
$(assume(*y \neq 1) : pc_6)$	$\langle *\langle y,0\rangle,9\rangle \neq 1$

Summary

- The described techniques were implemented in BLAST
- They can be invoked with option `-craig`

Thank you!

For Further Reading I

Henzinger, Jhala, Majumdar, McMillan
Abstractions from Proofs
In *ACM SIGPLAN-SIGACT Conference on Principles of Programming Languages, 2004*, 2(1):50–100, 2000.